Interior Power Flow

FOR THE INTERIOR POINT ALGORITHM

The Optimal Power Flow Problem

Optimizes the cost to generate power

- While satisfying
  - Network flow equations
  - Physical constraints
  - Operational constraints

Non-linear
Large scale
Static optimization problem

Both discrete and continuous variables

The Optimal Power Flow Problem

Voltage Vi = ei+ jfi

Four Objectives:

- Minimum generation cost
  - $\sum_{i=1}^{g}(c_{0i} + c_{1i}\, P_{gi} + c_{2i} P_{gi}^{2})$
- Minimum active power losses
  - $\sum_{i=1}^{n} \sum_{j=1}^{n} G_{ij}\left[(e_i - e_j)^2 + (f_i - f_j)^2\right]$
- Maximum power system loadability
  - $\max S = -\min(-S)$
- Minimum load shedding
  - $min \sum_{i=1}^{c} \phi_i P_{ci}$

V is voltage

Then for ith generator
C0i c01 and c2i are cost coeeficients (assume it’s quadratic)

Pgi is active output

Gij is conductance of branch for buses I and j

S is scalar of loadability factor

Theta I for ith load is percentage of load curtailment and Pci is active demand
Or demand response: reduce load for a brief crisis

Equality constraints

$$P_{gi} - (1 + S - \phi_i)P_{ci} - V_i^2 \sum_{j \in N_i} (G_{sij} + G_{ij}) + \sum_{j \in N_i} [(e_i e_j + f_i f_i)G_{ij} + (f_i e_j - e_i f_j)B_{ij}] = 0$$

$$Q_{gi} - (1 + S - \phi_i)Q_{ci} + V_i^2 \left[B_{si} + \sum_{j \in N_i} (B_{sij} + B_{ij}) \right] - \sum_{j \in N_i} [(e_i e_j + f_i f_j)B_{ij} + (e_i f_j - f_i e_j)G_{ij}] = 0$$

$$e_i^2 + f_i^2 - (V_i^{\mathrm{ref}})^2 = 0 \quad i = 1, \ldots, g.$$

Nodal active and reactive power balance equations

P and Q g are active and reactive power of generator connected at bus I
P and Q c are active and reactive demand of the load connected at bus I

Vi^2 = ei^2 + fi^2 so modulus of voltage

Bsi is shunt susceptance at bus I

Gij and Bij (Gsij and Bsij) are longitatadal (shunt) conductance and susceptance of the branch linking bsuesi and J

Ni set of buses connected by branches to the bus I

S=0 unless we deal with objective 3

Φϕ=0 I -1, …, c if load curtailment is not allowed as control variable

Also we might have additionall constaints line the one below: an equality constraints

Inequality constraints

$$(G_{ij}^2+B_{ij}^2)[(e_i-e_j)^2+(f_i-f_j)^2]\leq(I_{ij}^{\max})^2,\quad i,j=1,\ldots,n$$

$$(V_i^{\min})\leq e_i^2+f_i^2\leq(V_i^{\max})^2,\quad i=1,\ldots,n$$

$$P_{gi}^{\min}\leq P_{gi}\leq P_{gi}^{\max},\quad i=1,\ldots,g$$

$$Q_{gi}^{\min}\leq Q_{gi}\leq Q_{gi}^{\max},\quad i=1,\ldots,g$$

$$r_i^{\min}\leq r_i\leq r_i^{\max},\quad i=1,\ldots,o$$

$$x_i^{\min}\leq x_i\leq x_i^{\max},\quad i=1,\ldots,s$$

$$\phi_i^{\min}\leq \phi_i\leq \phi_i^{\max},\quad i=1,\ldots,c$$

Two inequalities
Opertaional: secure operation of system
Physical: limits of equipment

Constraints on branch rather than active power flow through branch (active, reactive and apparent power flow)

Operational on top

And physical on bottom
Active output limits for ith generator
And r for transformer
And theta for curtailment
And x for shunt bounds on reactance

R intervenes through Gsij, Bsij, Gij, and Bij
While x intervenes through Bsi only

Polar form Is more popular but both give very similar reulsts

Rectangle give quadratic intervenes: Hessian matrix is constant

Interior Point Method for OPF Problem

Iterative steps

All must be twice continuously differentiable, x is m-dimensional vector (with control variabes and state variables -> real and imaginary voltage at all buses). G is p-dimensional vector of function
And h is q dimensional vector of functions

Minimize min f(x) subject to g(x) = 0 and h(x) >= 0

Interior Point Method

$$\min f(\mathbf{x}) - \mu \sum_{i=1}^{q} \ln s_i$$

subject to:

$$\mathbf{g}(\mathbf{x}) = \mathbf{0}, \qquad \mathbf{h}(\mathbf{x}) - \mathbf{s} = \mathbf{0}$$

Add inequality constraints to objective function as logarithmic barrier terms to eliminate them

U is postivie scalar: the barrier parameter

Gradually decreased to zero as iterations progress

The whole thing depends on this

As u goes to zero, the solution x(u) converges to local optimum

Lagrangian

$$L_\mu(\mathbf{y}) = f(\mathbf{x}) - \mu\sum_{i=1}^{q}\ln s_i - \boldsymbol{\lambda}^T\mathbf{g}(\mathbf{x}) - \boldsymbol{\pi}^T[\mathbf{h}(\mathbf{x}) - \mathbf{s}]$$

Transform the equality constrained optimized into unconstrained one b defining the Lagrangian:

Delta and Pi are dual variables: the vectors of Lagrange multipliers

Y = [s,pi, theta, x]T

Lagrangian Derivatives

$$\begin{bmatrix} \nabla_{\mathbf{s}} L_\mu(\mathbf{y}) \\ \nabla_{\boldsymbol{\pi}} L_\mu(\mathbf{y}) \\ \nabla_{\boldsymbol{\lambda}} L_\mu(\mathbf{y}) \\ \nabla_{\mathbf{x}} L_\mu(\mathbf{y}) \end{bmatrix} = \begin{bmatrix} -\mu \mathbf{e} + \mathbf{S}\boldsymbol{\pi} \\ -\mathbf{h}(\mathbf{x}) + \mathbf{s} \\ -\mathbf{g}(\mathbf{x}) \\ \nabla f(\mathbf{x}) - \mathbf{J}_{\mathbf{g}}(\mathbf{x})^T \boldsymbol{\lambda} - \mathbf{J}_{\mathbf{h}}(\mathbf{x})^T \boldsymbol{\pi} \end{bmatrix} = \mathbf{0}$$

Set derivates to zero of the Lagrangian with respect to all unkowns ➔ KKT first order optimality conditions

S is diagonal matrix of slack variables

E = 1[,…,1]T
Delta f(x) is gradient of f

Jg(x) is Jacobian of g(x) and Hh(x) is Jacobian of h(x)

Iterations (Primal dual algorithm)

Iteration k =0, choose u(0) > 0

Initalize y(0) so that s(0) and pi(0) >0

Solve KKT (Karush-Kuhn Tucker) conditions for the Netwton direction delta y(k)

$$\mathbf{H}(\mathbf{y}^k)\begin{bmatrix}\Delta\mathbf{s}^k\\ \Delta\boldsymbol{\pi}^k\\ \Delta\boldsymbol{\lambda}^k\\ \Delta\mathbf{x}^k\end{bmatrix} = \begin{bmatrix}\mu^k\mathbf{e}-\mathbf{S}^k\boldsymbol{\pi}^k\\ \mathbf{h}(\mathbf{x}^k)-\mathbf{s}^k\\ \mathbf{g}(\mathbf{x}^k)\\ -\nabla f(\mathbf{x}^k)+\mathbf{J_g}(\mathbf{x}^k)^T\boldsymbol{\lambda}^k+\mathbf{J_h}(\mathbf{x}^k)^T\boldsymbol{\pi}^k\end{bmatrix}$$

where $\mathbf{H}(\mathbf{y}^k)$ is the second derivative Hessian matrix $(\partial^2 L_\mu(\mathbf{y}^k)/\partial\mathbf{y}^2)$.

H(y(k)) is second derivative Hessian matrix of

$$\alpha^k = \min\left\{1, \gamma \min_{\Delta s_i^k < 0} \frac{-s_i^k}{\Delta s_i^k}, \gamma \min_{\Delta \pi_i^k < 0} \frac{-\pi_i^k}{\Delta \pi_i^k}\right\}$$

$$\mathbf{s}^{k+1} = \mathbf{s}^k + \alpha^k \Delta \mathbf{s}^k \qquad \boldsymbol{\pi}^{k+1} = \boldsymbol{\pi}^k + \alpha^k \Delta \boldsymbol{\pi}^k$$

$$\mathbf{x}^{k+1} = \mathbf{x}^k + \alpha^k \Delta \mathbf{x}^k \qquad \boldsymbol{\lambda}^{k+1} = \boldsymbol{\lambda}^k + \alpha^k \Delta \boldsymbol{\lambda}^k$$

Third step

Determine max step length alpha k between 0 and 1 along Netwon direction delta y(k) such that s(k+1), pi(k+1) > 0

Gamma is between 0 to 1 is safety factor (to ensure positiviness of slack, usually set to 0.99995

And Update the solution

$$\max\left\{\max_i\{-h_i(\mathbf{x}^k)\},\ ||\boldsymbol{g}(\mathbf{x}^k)||_\infty\right\} \le \varepsilon_1$$

$$\frac{||\nabla f(\mathbf{x}^k) - \boldsymbol{J}_g(\mathbf{x}^k)^T\boldsymbol{\lambda} - \boldsymbol{J}_h(\mathbf{x}^k)^T\boldsymbol{\pi}^k||_\infty}{1 + ||\mathbf{x}^k||_2 + ||\boldsymbol{\lambda}^k||_2 + ||\boldsymbol{\pi}^k||_2} \le \varepsilon_1$$

$$\frac{\rho^k}{1 + ||\mathbf{x}^k||_2} \le \varepsilon_2$$

$$\frac{|f(\mathbf{x}^k) - f(\mathbf{x}^{k-1})|}{1 + |f(\mathbf{x}^k)|} \le \varepsilon_2$$

where $\rho^k = (\mathbf{s}^k)^T\boldsymbol{\pi}^k$ is called complementarity gap.

$$\mu^{k+1} = \sigma\frac{\rho^k}{q}$$

Check convergence : local optimality solution

If no convergence, then update the barrier parameter and repeat (on far right)

Set u(k+1) = sigma p(k)/q

Sigma is usually 0.2
K++ and go to step 2

Other methods

Dual Primal-dual (PD)

Predictor-corrector (PC)

Multiple Centrality Corrections (MCC)

Speed

Minimizing power loses

Table 1
Test systems summary

System	*n*	*g*	*c*	*b*	*l*	*t*	*o*	*s*
Nordic32	60	23	22	81	57	31	4	12
IEEE118	118	54	91	186	175	11	9	14
IEEE300	300	69	198	411	282	129	50	14

Table 2
Number of iterations to convergence and CPU times

Interior-point algorithm	Nordic32		IEEE118		IEEE300	
	Iters.	Time	Iters.	Time	Iters.	Time
PD	11	0.31	13	0.62	16	1.63
PC	8	0.26	10	0.58	12	1.44
MCC	6	0.24	7	0.52	10	1.48

PC with 512 MB RAM and 1.7GHZ Pentium IV
Sum of power loses: active power loses

Cotntrol: slack generator active power, reactive power, transformer ratio and shunt reactiance

Quality bus power balance

Inequality for generators reactive power, voltage magnitude +-0.05PU , transformer with controllable ratio, shunt reactance

The active losses at the optimum are: 151.74 MW for the Nordic32 system, 116.52 MW for the IEEE118 system and 386.6 MW for the IEEE300 system, respectively. This corresponds, respectively, to 7.90%, 12.31%, and 5.37% of less active losses than in the base case

Speed 2

Minimizing overall generation costs

Table 5
Number and type of active constraints

System	Active constraints						
	P_g	Q_g	I	V	r	x	Total
Nordic32	15	1	5	26	0	4	51
IEEE118	3	8	3	25	0	5	44
IEEE300	3	26	4	59	3	2	97

Table 4
Number of iterations to convergence and CPU times

Interior-point algorithm	Nordic32		IEEE118		IEEE300	
	Iters.	Time	Iters.	Time	Iters.	Time
PD	21	0.82	18	1.10	23	3.02
PC	13	0.61	11	0.80	15	2.33
MCC	12	0.71	10	0.90	11	2.16

Cotntrol: slack generator active power, reactive power, transformer ratio and shunt reactiance

Equality: bus active and reactive power flow

Inequality all above plus voltages magnitude and branch currents

Other things you can do

Maximizing Power System loadability

Minimizing the amount of load shedding

Increase loads proportionally over their base consumptions
Increase covered by a single slack generator

Minimize the amount of load shedding for an infeasible power system situation such that an equilibrium point is restored
Power shedding is done at constant power factor less than 0.1 and higher than 0
Slack generator compensates active power imbalance

Challenges

Security Constrained OPF (SCOPF)

Handles together base constraint and steady state security constraints relative to credible contingenecies
-> extension of OPF
Two types: preventive and corrective SCOPF
High dimenstinality of problem for many contingency cases and large power systems

Define harmfule contingencies only

Questions

Thank you for listening

References

Capitanescu, Florin, et al. "Interior-point based algorithms for the solution of optimal power flow problems." *Electric Power systems research* 77.5 (2007): 508-517.

Vargas, Luis S., Victor H. Quintana, and Anthony Vannelli. "A tutorial description of an interior point method and its applications to security-constrained economic dispatch." *Power Systems, IEEE Transactions on* 8.3 (1993): 1315-1324.