A NYQUIST FOLDING ANALOG-TO-INFORMATION RECEIVER

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ABSTRACT

Many radar and communications applications require detection and estimation of signal information across an extremely wide radio frequency (RF) bandwidth. In practice, however, direct digitization of this broadband RF environment is problematic. Physical limitations in analog-to-digital converter (ADC) technology restrict the total bandwidth that can be digitized, as well as the ability to digitize high RF signals directly. This paper describes a novel "analog-to-information" receiver, motivated by recent developments in Compressed Sensing (CS), which overcomes both of these challenges in certain settings. The proposed receiver performs frequency modulated pulsed sampling at sub-Nyquist/Shannon rates to compress a broadband RF environment into an analog interpolation filter. The RF sample clock modulation induces a Nyquist-zone dependent frequency modulation on the received signals, allowing separation and recovery of the signal information from a sparse broadband RF environment.

Index Terms— Compressive sensing, microwave receivers, analog-digital conversion.

1. INTRODUCTION

With continued advances in digital signal processing (DSP) technology, the analog-to-digital converter (ADC) is the limiting factor in a number of applications, including some radar and communications signal applications that require information processing across extremely wide RF bandwidths. ADCs are constrained both in digital bandwidth (i.e., sample rate) [1] and in analog bandwidth (i.e., the ability to directly digitize high RF bands) [2]. A recent concept in the signal processing literature is to sample the environment based on the information rate rather than the bandwidth. When the signal environment is relatively sparse, the reduction in sample rate may be very large. It is shown in [3], for example, that Dirac sequences may be sampled at a finite rate of innovation (FRI) – even with extra sampling to give robustness against noise, this is still far less than the Nyquist/Shannon rate for ultra wideband impulse-like signals. Although these initial results apply to a limited class of signals, recent results in compressed sensing (CS) have shown more generally that the information from a signal can be captured with far fewer measurements than the traditional Nyquist/Shannon criteria, as long as the signal has a sparse (or nearly sparse) representation in some basis or frame [4], [5]. One of the more surprising aspects of CS (also called compressive sampling), is that the prescribed measurements do not require *a priori* signal knowledge beyond the basic sparsity or compressibility assumption. Thus it is possible, at least in principle, to design a universal CS measurement system that can be used to encode a wide range of signal types. The idea of sampling based on information rate, rather than the Shannon bandwidth criteria, suggests a new approach denoted *analog-to-information* (A-to-I) as an alternative to conventional ADCs or digital receivers [6], [7].

The applicability of CS theory to practical RF receivers has been limited to date. Original CS theory described the recovery of sparse (and perhaps very high-dimensional) vectors from a set of observations in the form of projections of a static signal onto random basis vectors. This model is ill-suited to RF applications for several reasons. First, because of the time-varying nature of RF signals, the assumption of a persistent static signal is not valid. Second, the discrete nature of this model implicitly assumes that the signal has already been sampled, which leads back to the original problem of the ADC limitations. Nonetheless, some promising approaches for practical receivers have been proposed, including discrete-time random filters [8], [9], random sampling [10], and random demodulation [11]-[13]. Note that A-to-I architectures based on either discrete time random filtering or random demodulation still require Nyquist-rate components relative to the maximum analog frequency at the receiver front-end, thus limiting the applicability of these approaches for direct processing of high RF signals. On the other hand, A-to-I approaches based on non-uniform sampling are feasible for direct RF implementation without the need for Nyquist-rate components. In this paper we describe a novel approach that uses structured non-uniform sampling, rather than random sampling, to implement a direct RF A-to-I receiver that is effective at recovering signals that have a sparse frequency-domain representation [14]. Among the benefits of the proposed receiver is its greatly simplified signal recovery compared to random sampling.

2. COMPRESSIVE SENSING

Consider the problem of recovering unknown, real, length N vectors that are sparse in some specified basis or dictionary. Let x be such a vector, and assume it is K-sparse, meaning that it can be expressed as a linear combination of K elements of the basis set $\{\psi_i\}$; for example, $x = \sum_{i=1}^{k} \alpha_i \psi_i$.

Recovery of x is, of course, possible using traditional methods – for example, by sampling each entry of x. The problem with this approach is that sampling at such a rate could be physically impossible, and even if it were possible we would expend quite a bit of energy encoding bits that will eventually be discarded in the final representation. The question of whether there is a way to “just measure directly the part that won’t end up being thrown away” was posed and investigated in [5]. The answer is yes, and compressive sensing describes a collection of methods by which x can be recovered using a minimal number of measurements.

If the correct basis elements were known prior to sampling, one could simply observe the K relevant entries of the signal in the representation $\{\psi_i\}$. But, the lack of prior signal knowledge makes this an impossible task. Instead, the CS approach prescribes collecting samples that are projections of the unknown signal onto elements from a second basis set $\{\phi_i\}$ that is *incoherent* with $\{\psi_i\}$. By incoherent, we mean that a sparse representation of any element of the basis set $\{\phi_i\}$ does not exist using the basis vectors $\{\psi_i\}$ and visa-versa [4], [5]. A collection of such observations can be described by the linear model $y = \mathbf{\Phi} x$, where y is a length M vector and $K < M << N$.

One of the significant results of CS theory is that incoherent projections can be obtained without prior knowledge of the signal – in particular, random vectors will be incoherent with any fixed basis with high probability. Reconstruction of the unknown signal does require knowledge of the basis in which the signal is sparse, and can be accomplished by solving an l_1 minimization problem, finding the estimate for x in the underdetermined set of linear equations:

$$\begin{aligned} \hat{\alpha} &= \arg\min \|\alpha\|_1 \quad s.t. \quad y = \Phi x = \Phi \Psi \alpha \\ \hat{x} &= \Psi \hat{\alpha}. \end{aligned} \tag{1}$$

On average, CS succeeds in recovering K sparse vectors when the number of observations greater than a small constant times $K \log(N)$ [4], [5].

Since CS involves discrete-time observations, one of the goals of practical A-to-I receivers is to extend and apply discrete time CS concepts to an analog continuous time signal environment. The discrete-time random filter approach proposed in [8] can be recast in the CS framework, where the effective acquisition matrix has a Toeplitz structure. Theoretical guarantees for CS using such constructs were first established in [9]. A random arithmetic sampling progression is investigated in [10], where information recovery for locally Fourier sparse signals is performed via the sparsogram – a fast iterative greedy pursuit algorithm that includes computing a non-uniformly sampled fast Fourier transform algorithm on the sampled residual. Random demodulation methods were investigated in [11]-[13], where the observation model prescribes modulating the incoming signal by a random sequence, integrating the modulated signal, and subsampling the output to obtain the low-dimensional data. Our novel approach, described below, relies on non-uniform chirped sampling for simplified information recovery compared to other A-to-I approaches, and allows sampling high analog input frequencies without the need for high-speed components operating at the Nyquist rate for the maximum analog input frequency.

3. NYQUIST FOLDING A-TO-I RECEIVER

The primary challenge in reconstructing a signal from its samples is that many different signals could possibly give rise to the same set of samples. For example, uniformly subsampling a signal can lead to aliasing, preventing recovery of the original signal frequencies. The novel sampling scheme proposed here overcomes this issue by imposing a *frequency-dependent* signature on each component of the original signal, from which the original signal component frequencies can be obtained.

We now examine this proposed A-to-I receiver architecture, which folds multiple Nyquist zones into a narrow bandwidth prior to ADC conversion, in more detail. The **Ny**quist **F**olding **R**eceiver (NYFR), shown in Figure 1, uses a wideband pre-select filter $H(\omega)$ rather than a standard anti-aliasing filter prior to sampling; thus allowing multiple Nyquist zones to be sampled and subsequently folded into a continuous time analog interpolation filter. The RF sample clock is modulated about a carrier to provide a non-uniform sample rate, as described here. Following the discussion in [16], the phase of the RF sampling clock may be viewed as an occurrence function for a monotonically increasing function $\varphi(t)$ – the samples are taken as $\varphi(t)$ crosses multiples of 2π. In other words, sample times correspond to zero crossing rising voltage times of $\sin(\varphi(t))$, implying an instantaneous sample rate of $\varphi'(t)$ samples per second. Other than the fact that the sample clock is modulated and the filter $H(\omega)$ allows aliasing, the front-end portion before the ADC follows the standard impulse sampling paradigm [17]. Note that this architecture also provides for wide analog input bandwidth without the need for high speed sample and hold circuits or Nyquist rate components [15].

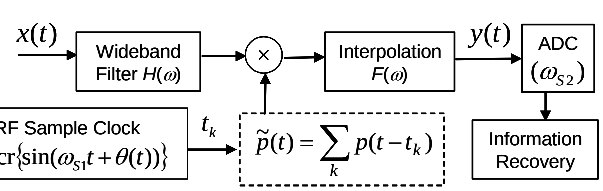

Figure 1: Nyquist Folding A-to-I Receiver Architecture

We focus on the case where $F(\omega)$ is an ideal low-pass filter with cutoff $\omega_{S1}/2$ and where $\varphi(t)$ represents a narrow-band phase (or frequency) modulation centered at ω_{S1}:

$$\varphi(t) = \omega_{S1} t + \theta(t). \qquad (2)$$

For this narrowband frequency modulated RF sample clock, if the input is a narrowband signal with center frequency ω_c and information modulation $\psi(t)$

$$x(t) = \cos(\omega_c t + \psi(t)), \qquad (3)$$

the normalized interpolation filter output will be

$$y(t) \approx \cos(|\omega_c - \omega_{S1} k_H| t + \beta\psi(t) - M\theta(t)), \qquad (4)$$

where k_H=round(ω_c / ω_{S1}), β=sgn(ω_c - ω_{S1} k_H), and $M=\beta\, k_H$. The value k_H is the harmonic in the Fourier series of the pulse train that corresponds to the interpolation filter output, β is negative for spectrally reversed bandpass sampled signals from odd Nyquist zones N_S (see Figure 2), M is the resulting modulation scale factor, and $|\omega_C - \omega_{S1} k_H|$ is the intermediate frequency after bandpass down-conversion sampling [18]. A derivation of this result is outlined in the Appendix.

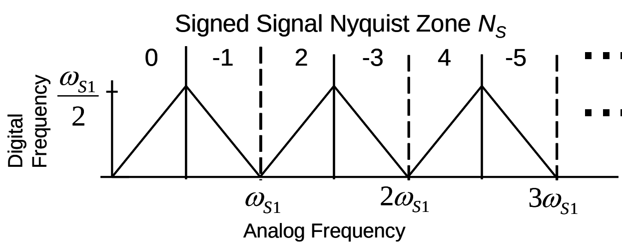

Figure 2: Bandpass Sampled Signal Nyquist Zones

In essence, the result in (4) implies that the received signal has an induced modulation $M\theta(t)$ of the same form as the RF sample clock modulation $\theta(t)$, with a modulation scale factor M and orientation β depending on the signal Nyquist zone N_S. Thus even though multiple signals from different Nyquist zones may alias into the same band, the information from the different signals, including the original RF, can still be recovered. We take advantage of the fact that the added modulation is different for each Nyquist zone so that the folded signals are separable when the signal environment is relatively sparse. Note that the continuous time interpolation filter allows the RF sample rate to be de-coupled from the ADC sample rate so that the ADC may sample at a uniform rate. This feature helps to simplify the clocking of the digital signal processing (DSP), including local data movement between ADC and DSP.

With narrowband frequency modulated (FM) RF sampling in the NYFR architecture of Figure 1, processing can be performed directly on the folded data without solving a computationally complex optimization. For example, if the sample clock is a FM continuous wave (FMCW) periodic chirped signal, received narrowband signals will all have the same chirp pattern in the time-frequency plane, with center frequencies depending on bandpass sampling translation and frequency modulation scale factors depending on originating Nyquist zone. Signal recovery may be performed by de-modulation as opposed to traditional l_1 minimization approaches, as in (1).

3. ILLUSTRATIVE SIMULATION EXAMPLE

In this section, we illustrate the operation of the NYFR using a simulation with four signals. Figure 3 illustrates a Matlab example of the NYFR with an ideal low pass interpolation filter and a sinusoidal FMCW RF sample clock. The top panel frequency-time plot shows four narrowband signals sampled at 20 Gsps – signal A is at 900 MHz; signal B is at 1.9 GHz; signal C is at 3.2 GHz; signal D is at 8.4 GHz. The middle panel shows the RF sample clock varying from 1950 to 2050 Msps over a 2.5 micro-second window, with an average sample rate of 2.0 Gsps. The bottom panel shows the folded narrowband signals at the output of the interpolation filter with unique Nyquist zone dependent modulation for each signal.

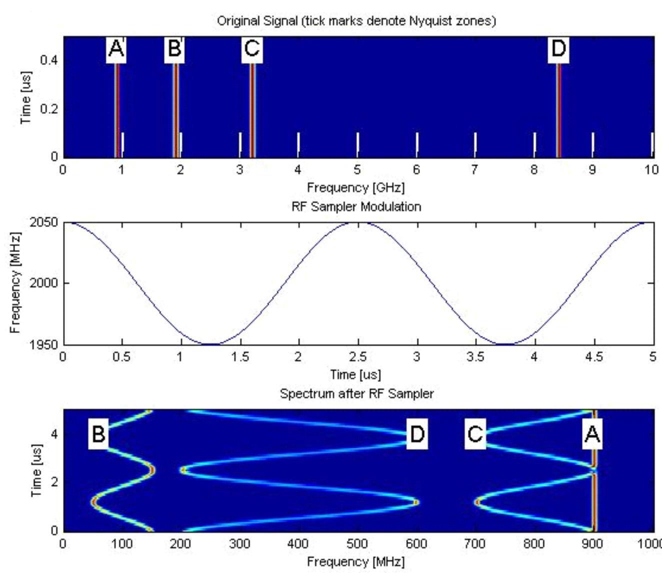

Figure 3: Sinusoid FMCW Sample Clock Signal Example

Table 1 shows the signal Nyquist zone, sampling harmonic (k_H), folded intermediate frequency (IF), and modulation bandwidth for each of the four signals using (4) and signed Nyquist zones N_S corresponding to Figure 2.

Table 1: Illustrated Example Calculated Values

Signal	N_S	k_H	M	IF (MHz)	$M\Delta F$ (MHz)
A	0	0	0	900	0
B	-1	1	-1	100	-100
C	-3	2	-2	800	-200
D	8	4	4	400	400

5. LAB TEST EXAMPLE

In this section, we present experimental results using a DCSM 7620 sampling device from Picosecond Pulse Labs [19] with a nominal sample aperture of about 20 picoseconds. In this example, we consider three narrow-band tones – Signal A is at 7.65 GHz; Signal B is at 17.22 GHz; and Signal C is at 32.6 GHz. The interpolation filter bandwidth is about 850 MHz. The output of the interpolation filter is uniformly sampled by an Atmel 10-bit ADC at 2 Gsps. For this experiment, we consider two cases: Uniform RF sampling at 2 Gsps and modulated RF sampling using an FMCW clock waveform (similar to the prior illustrative example) with an average sample rate F_{S1} = 2000 Msps, ΔF = 5 MHz, and modulation period = 2 μsec.

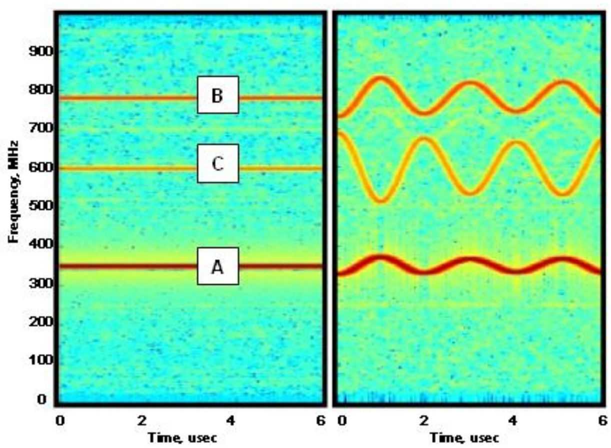

Figure 4: Multi-Signal Lab Test Example

The left panel of Figure 4 shows the results after uniform sampling at 2 Gsps. As predicted by conventional bandpass sampling, signals A, B, and C alias (or fold) to 350 MHz, 780 MHz, and 600 MHz respectively. For this case, recovery of the original frequencies is not possible. The right panel of Figure 4 shows the resulting time-frequency plot with the FMCW modulated RF sample clock. The corresponding calculated values for this lab test are presented in Table 2 below. From this example, we can see that the signals fold to the correct locations and that the induced modulation bandwidth and modulation orientation corresponds to the expected values listed in Table 2. With the Nyquist-zone dependent induced modulation, it is possible to determine the original RF; it is also possible to remove the induced modulation to recover the original signal waveform.

Table 2: Multi-Signal Lab Test Calculated Values

Signal	N_S	k_H	M	IF (MHz)	$M\Delta F$ (MHz)
A	-7	4	-4	350	-20
B	-17	9	-9	780	-45
C	32	16	16	600	80

6. CONCLUSIONS & FUTURE RESEARCH

The Nyquist Folding Receiver is an "analog-to-information" receiver motivated by compressive sensing that performs frequency modulated pulsed sampling directly at the RF to allow for unambiguous recovery after compressing multiple Nyquist zones into an analog interpolation filter. The RF sample clock modulation induces a Nyquist-zone dependent frequency modulation on the received signals that can be measured and removed when the RF signal environment is relatively sparse and the folded signals do not overlap significantly. With this architecture, it is possible to recover signals without performing a computationally complex l_1 minimization. In some applications, the folded signal parameters may be detected and measured directly in the compressed space, further reducing the computational complexity. Although we have limited the discussion to narrow-band (Fourier-sparse) signals in this paper, the NYFR is applicable to a broader class of signals. Future research will examine NYFR performance using Restricted Isometry Properties of CS matrices formed from the modulated RF sampling, including extensions to signals that are not Fourier-sparse.

APPENDIX

In this section, we outline a derivation for the Nyquist Folding Receiver at the interpolation filter output *y(t)*. Note that the pulse train $\tilde{p}(t)$ can be expressed as a convolution of a Dirac sequence with a pulse template $p(t)$. Using the Dirac scaling property we have

$$\tilde{p}(t) = p(t) * \varphi'(t)\sum_k 2\pi\delta\left(\varphi(t) - 2\pi k\right), \qquad (5)$$

where $\varphi(t)$ is the phase function of the sampling oscillator. Using the identity

$$2\pi\sum_k \delta\left(z - 2\pi k\right) = \sum_k e^{jkz}, \qquad (6)$$

and following [20] with $z = \omega_{S1}t + \theta(t)$, we have

$$\begin{aligned} \tilde{p}(t) &= p(t) * \left(\omega_{S1} + \theta'(t)\right)\sum_k e^{jk\left(\omega_{S1}t + \theta(t)\right)} \\ &\approx p(t) * \omega_{S1}\sum_k e^{jk\left(\omega_{S1}t + \theta(t)\right)}, \end{aligned} \qquad (7)$$

where the approximation in (7) assumes that the RF sample clock modulation is narrowband so that $\omega_{S1} >> \max|\theta'(t)|$. The impact of this approximation can be appreciated by noting that the output magnitude is proportional to the sample rate. As the sample rate changes, the output amplitude varies accordingly. For example, if the sample rate varies from 1950 Msps to 2050 Msps (as described in the Matlab example in Section 3) the approximation results in an amplitude variation of $20\log_{10}(2050/1950)$, or slightly greater than 0.4 dB.

We now take the Fourier transform of the pulse train:

$$\begin{aligned}\tilde{P}(\omega) &\approx P(\omega)\omega_{S1}\sum_k\left(\delta(\omega-k\omega_{S1})*\Im\left\{e^{jk\theta(t)}\right\}\right)\\ &\approx \omega_{S1}\sum_k P(k\omega_{S1})\left(\delta(\omega-k\omega_{S1})*\Im\left\{e^{jk\theta(t)}\right\}\right)\\ &= \sum_k T_k(\omega-k\omega_{S1}),\end{aligned} \quad (8)$$

where the narrow-band modulation term $T_k(\omega)$ is given by:

$$T_k(\omega)=\omega_{S1}P(k\omega_{S1})\Im\left\{e^{jk\theta(t)}\right\}. \quad (9)$$

The additional approximation in (8) assumes that the sample aperture is short so that the Fourier transform of the pulse *$P(\omega)$* is approximately constant over the frequency range ω where $k[\omega_{S1}-\min(\theta'(t))]<\omega<k[\omega_{S1}+\max(\theta'(t))]$.

Using the notation of Figure 1, for input signals $x(t)$ that fall within the passband of the pre-select filter $H(\omega)$, the output $y(t)$ of the interpolation filter in the frequency domain is given by:

$$\begin{aligned}Y(\omega) &= \left(\left(X(\omega)H(\omega)\right)*\tilde{P}(\omega)\right)F(\omega)/2\pi\\ &\approx \left(X(\omega)*\sum_k T_k(\omega-k\omega_{S1})\right)F(\omega)/2\pi\\ &= \left(\sum_k X(\omega-k\omega_{S1})*T_k(\omega)\right)F(\omega)/2\pi\\ &= (X_L(\omega-k_H\omega_{S1})*T_{k_H}(\omega)+\\ &\qquad X_R(\omega+k_H\omega_{S1})*T_{-k_H}(\omega))/2\pi.\end{aligned} \quad (10)$$

where X_L and X_R represent the left and right hand sides of the Fourier transform respectively (i.e., $X_L(\omega) = X(\omega)$ for $\omega < 0$ and 0 otherwise; similar for $X_R(\omega)$). In (10), we assume that the signal and modulation $T_k(\omega)$ are narrow enough in bandwidth so that the shifted left and right hand terms fall completely within the interpolation filter $F(\omega)$ for the bandpass sampling harmonic k_H. In particular, for the case where $x(t)$ is a narrow-band signal as given by (3), we have

$$\begin{aligned}X_L(\omega) &= \delta(\omega+\omega_C)*\Im\left\{e^{-j\psi(t)}\right\}/2\\ X_R(\omega) &= \delta(\omega-\omega_C)*\Im\left\{e^{j\psi(t)}\right\}/2.\end{aligned} \quad (11)$$

Substituting (11) into (10) results in:

$$\begin{aligned}Y(\omega) &= (\delta(\omega+\omega_C-k_H\omega_{S1})*\Im\left\{e^{-j\psi(t)}\right\}*T_{k_H}(\omega)\\ &\quad+\delta(\omega-\omega_C+k_H\omega_{S1})*\Im\left\{e^{j\psi(t)}\right\}*T_{-k_H}(\omega))/4\pi.\end{aligned} \quad (12)$$

Taking the inverse Fourier transform yields the desired time-domain result.

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