Consider the second order system given by
\[ \ddot{p} + 2\xi\omega_0 \dot{p} + \omega_0^2 p = \omega_0^2 f \]

1. Let the output be \( y = p \). Determine the transfer function from \( f \) to \( y \).

2. Determine the state space representation of the above input-output system in the form \( \dot{x} = Ax + Bu, \quad y = Cx + Du \).

3. Sketch a Analog Computer Simulation model and implement it in Simulink

4. Assume that \( p(0) = \dot{p}(0) = 0 \) and simulate the step-response of the system with \( \xi = 0.2, \omega_0 = 1\text{rad/s} \).

5. Explore the command \texttt{ss} and \texttt{step} in Matlab. Note that the command \texttt{step} operates on an object of type \texttt{sys}. Use these commands to obtain the step response using Matlab and compare it with the step response obtained using Simulink
2. (a) Consider a system with the following input \((u(t))\)-output \((y(t))\) relation:

\[
\ddot{y} + 10\dot{y} + 9y = u
\]

i. Find the transfer function that describes this system
ii. Give a state space representation of this system.

(b) Consider a system with the transfer function

\[
G(s) = \frac{Y(s)}{U(s)} = \frac{1}{s^2 + 0.1s + 4}.
\]

i. Give the ordinary differential equation that describes the relation between the input \(u\) and the output \(y\) of the system.
ii. Give a state space representation of the same system.

(c) Consider a system with the following state space representation

\[
\begin{pmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{pmatrix} = \begin{pmatrix}
0 & 1 \\
-1 & -1
\end{pmatrix} \begin{pmatrix}
x_1 \\
x_2
\end{pmatrix} + \begin{pmatrix}
0 \\
1
\end{pmatrix} u
\]

\[
y = \begin{pmatrix}
1 & 1
\end{pmatrix} \begin{pmatrix}
x_1 \\
x_2
\end{pmatrix} + 2u
\]

i. Find the ordinary differential equation that describes this system
ii. Find a transfer function that describes this system (Hint: Find \(X_1(s)\) and \(X_2(s)\) in terms of \(U(s)\) and substitute in terms of \(Y(s)\)).
3. (a) Consider the transfer function \( \frac{V(s)}{U(s)} = \frac{1}{s^2 + 4s + 1} \). Find a state space representation for this system.

(b) Let \( y = \ddot{y} + 3\dot{y} + v \).

   i. Write \( y(t) \) in terms of the states in (3a)
   ii. Find the transfer function \( \frac{Y(s)}{V(s)} \)

(c) Use parts (3a) and (3b) to determine the state space representation of the system described by

\[
\ddot{y} + 4\dot{y} + y = \ddot{u} + 3\dot{u} + u.
\]

4. The model used to design a cruise controller gives the following relation between throttle position \( u \) and velocity \( v \)

\[
m\ddot{v} + cv = ku.
\]

At a particular driving condition with one driver (of weight 70 Kg) the parameters are \( m = 1600 \text{ [kg]} \), \( c = 32[kgm/s] \) and \( k = 1200[N] \). Give the transfer function of the system. Also give the transfer function when there are four passengers 70 Kg each in the car.
5. Consider the schematic of a DC motor shown in the Figure above. A common actuator in control systems is a DC motor. The electric circuit of the armature and the body diagram of the rotor are shown in Figure. Consider as input the voltage $V(t)$ and as output the angular position of the load $\theta$. The torque applied by the motor is $T_1 = K_e i$, whereas the emf is $e = K_e \dot{\theta}_1$, $i$ designating current. We assume that the shaft is flexible and denote by $\theta_1$ and $\theta$ the angular position of the two ends. We assume a simple “mass-less spring” type of model for the shaft, i.e., that the torque values $T_1$ applied to the shaft by the motor, and $T$ applied to the load by the shaft are equal, i.e.,

$$T = T_1 \quad \text{and that } \theta_1 - \theta = \alpha T.$$

The electromechanical part of the system is modeled by the equations

$$L \frac{di}{dt} + Ri + K \dot{\theta}_1 = V$$

$$K i = T_1$$

$$J \ddot{\theta} + b \dot{\theta} = T.$$

Find the transfer function representation of the DC motor system $\frac{\Theta(s)}{V(s)}$ in terms of $J, b, K, L, R, \alpha$. 