ReSCALE: Recalibrating Sensor Circuits for Aging and Lifetime Estimation under BTI

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Abstract—Bias temperature instability (BTI) induced delay shifts in a circuit depend strongly on its operating environment. While sensors can capture some operating parameters, they are ineffective in measuring vital performance shifts due to changes in the workloads and signal probabilities. This paper determines the delay of an aged circuit by amalgamating more frequent measurements on ring-oscillator sensors with infrequent online delay measurements on a monitored circuit to recalibrate the sensors. Our approach reduces the pessimism in predicting circuit delays, thus permitting lower delay guardbanding overheads compared to conventional methods.

I. INTRODUCTION

Bias temperature instability (BTI) is an aging effect that causes the magnitude of the threshold voltage in nanometer-scale circuits to increase under temperature and voltage stress. BTI-induced aging can be partially reversed when the stress is removed, but the general delay trend over long periods shows a degradation over time.

It is crucial to estimate the extent of aging so that remedial techniques can be applied to ensure reliable operation over the lifetime of a circuit. These schemes may be deployed at the presilicon as well as at the post-silicon stage of design. Of necessity, presilicon design must be predicated on the worst-case workload for the circuit so that it is guaranteed to work under all operating conditions. This involves the application of pessimistic guardbands whose power overheads may be excessive and unnecessary for a large fraction of parts in the field. This pessimism could, in principle, be reduced by the use of signal probabilities (SPs) [1] that mimic the operating environment, but this requires foreknowledge of the average workload in real operating conditions in the field, which is often unavailable.

Specifically, a circuit that is designed to meet lifetime requirements under a specific average workload may well be used in a very different way by the customer, with very different SP and aging characteristics. As a result, chip design teams often treat SP-based methods with skepticism. Post-silicon techniques, on the other hand, rely on data from surrogate aging sensors [2]–[4], such as ring oscillators, to apply just enough adaptive compensation to mitigate the effect of aging [5], [6]. To a limited extent, they may successfully capture the environment faced by the circuit, e.g., if they are placed close to the circuit and have a similar connection to the power grid, they can capture the thermal and supply voltage environment. However, aging sensors are surrogates and cannot reflect aging in the circuit with accuracy, since the types of gates in the circuit and the signal stressing patterns and SPs for the two circuits are different.

As an example, let us consider the aging trends of representative circuits of the IWLS 2005 benchmark suite [7] over sets of pseudo-random input probabilities. To reflect signal distributions in real circuits, where the input SPs are typically biased towards 0 or 1 as against the unrealistic “academic” assumption of SP=0.5, these probabilities are generated from a bimodal distribution with peaks at SP=0.1 and SP=0.9 (in consistence with [8]). Each Monte Carlo (MC) simulation corresponds to a sample of these input SPs, propagated throughout a circuit to generate SPs at internal nodes, which are translated into a delay degradation number for each gate. A static timing analysis (STA) run is performed using these degraded gate delays to determine the temporal degradation in the circuit delay. We perform 500 such simulations for each of the circuits.

The pessimistic delay for each circuit is obtained by assuming the worst-case workload at every gate input, as in [9]–[11]. Such pessimistic delays, typically used to define a presilicon aging margin, may result in high power/area overheads during optimization [1], [5].

To quantify this pessimism, for each MC run of a circuit, we define the speed wastage factor, SWF(i, t), for the ith run as:

$$SWF(i, t) = \frac{f'(t) - f_{pess}(t)}{f'(t)}$$  \hspace{1cm} (1)

where f'(t) and f_{pess}(t) are, respectively, the frequencies at time, t, corresponding to the ith MC sample and the worst-case workload case. Since the maximum value of SWF(i, t) occurs at t = t_f, we plot the average, minimum and maximum values of the vector, SWF(t_f), whose ith element is SWF(i, t_f), in Fig. 1 (left). For further elaboration, the histogram of the SWF(t_f) for a specific circuit, wb_dma, is also shown in Fig. 1 (right) whose mean is 10.38%. In other words, if the aging sensor is calibrated using a pessimistic worst-case probability, this circuit is operated at a frequency about 10% slower than its true capability, consuming unnecessary power/area overheads.

The root cause of the pessimism seen in Fig. 1 is that the prediction is associated with (a) a presilicon characterization and (b) uses a surrogate aging sensor, which has inherent inaccuracies. The worst-case aging trend for a circuit is typically predicted by assuming worst-case stress on all gates. While such a method is guaranteed to be pessimistic, the pessimism may be too large. Some works have attempted to overcome this by using a worst-case stress probability of 0.95 instead of 1.0 on each gate [10], but this is purely empirical. Precise aging information is only obtainable from expensive post-silicon aging measurements performed directly on the circuit [12]–[14] instead of using surrogate sensors. The objective of this work is to build an efficient and precise scheme for diagnosing circuit delay.
degradation due to aging. Although the scheme can also be extended to any aging mechanisms such as hot carrier injection (HCI) that causes threshold voltages to change over time, our work focuses on BTI, which is the dominant aging mechanism in most products.

Our approach blends the simplicity and low measurement overhead of surrogate sensors with the accuracy of direct measurement. Our scheme avoids the large pessimism gap shown in Fig. 1 by recalibrating a set of surrogate sensors based on direct measurements on the circuit to diagnose its actual aging. These measurements are performed only occasionally, thus controlling the high overhead of runtime measurements. Furthermore, we develop a new theory that maps the results of direct measurement to the aging of the surrogate sensor, and propose a framework to recalibrate the surrogate sensors based on measurement data. We demonstrate significant improvement in the speed wastage factor on representative benchmarks and compare our approach with the traditional pessimistic one (Table I).

II. OVERVIEW OF THE SCHEME

Our scheme uses an initial calibration of the aging sensor obtained under presilicon worst-case aging of the circuit. Following the runtime measurements on the circuit at a set of measurement instants, the sensors are recalibrated to reflect the true past workload of the circuit, and refine its future aging estimate. The aging estimate beyond any measurement instant must assume the worst stressing SPs for the circuit since the data from such instant only provides information about the past and cannot be used to predict its future workload.

Consider a circuit under test (CUT) with a lifetime $t_f$, and a set of measurement instants, where the $i$th instant is denoted by $t_m$. Let,

- $D_{wc}(t)$ be the worst-case delay curve obtained from presilicon analysis, assuming all gates to be maximally stressed.
- $D_{act}(t_m)$ be the measured delay obtained at $t_m$, corresponding to the actual delay curve of the CUT under its true workload, which is impossible to predict at the presilicon stage. The workload depends on factors such as circuit activity, whether or not the circuit was in sleep mode, and usage statistics.
- $D_{est}(t)$ be the estimated aging curve using our approach. This curve follows $D_{wc}(t)$ until the first measurement instant, and based on $D_{act}(t_m)$, it is recalibrated and modified beyond $t_m$.

![Fig. 2. CUT delay measurement for sensor recalibration: an example.](image)

As a representative outcome of our scheme, consider Fig. 2, where we conceptually depict $D_{wc}(t)$, $D_{act}(t)$ and $D_{est}(t)$ for a single measurement instant, $t_m$. The multiple curves for $D_{act}(t)$ beyond $t_m$ emphasize the fact that multiple possible future workload scenarios may exist for the CUT based on circuit usage. The $D_{act}(t)$ curve is chosen so that it provides a guaranteed upper bound on the delay beyond $t_m$, by assuming the worst-case workload for the CUT beyond $t_m$, thus catering to the worst-case scenarios.

To realize our scheme, any functional block must include the aging sensors and the sub-blocks for aging estimation and runtime measurement amidst multiple CUTs as shown in Fig. 3.

![Fig. 3. A functional block equipped with aging estimation tools.](image)

Fig. 3: Circuit #1
- ROSCs (aging sensors) interspersed within four circuits.
- Stores calibration factors & CUT delays at measurement instants.

Fig. 3: Circuit #2
- LUT
- Test patterns
- Stores test patterns to excite the critical paths during infrequent true delay measurements of the CUT.

Fig. 3: Circuit #3
- LUT
- Test patterns

Fig. 3: Circuit #4
- LUT
- Test patterns

#### A. Modeling BTI-induced delay degradation

The effect of BTI is to increase the absolute value of the threshold voltage, $V_{th}$, of both PMOS (by Negative BTI or NBTI) and NMOS (by Positive BTI or PBTI) devices, which causes circuit delays to increase with time. The two most common models used to describe BTI aging are the Reaction-Diffusion (RD) model [16] and the Charge-Trapping (CT) model [17]. Each of these models describes temporal degradation in threshold voltage, $\Delta V_{th}(t)$, as:

$$\Delta V_{th}(t) = c h(\xi)f(t)$$

(2)
where \( f(t) \sim t^n, n \sim 0.1 - 0.4 \) (by RD model); \( f(t) \sim \log t \) (by CT model), \( h(\cdot) \) is a function of the stressing SP (which captures BTI recovery effects), \( \xi \), where for PMOS [NMOS], \( \xi \) is the probability of signal being low [high], and \( c \) depends on the temperature and supply voltage. The function, \( f(t) \), can be assumed to be the same for both PMOS and NMOS based on [18]. We note that \( f(t) \) is a monotonically increasing function, and we use this property later.2

The delay of a logic gate undergoing BTI aging is given by \( D(t) = D(0) + S \Delta V_{th}(t) \), where \( D(0) \) is the nominal delay of the gate and \( S \) is its delay sensitivity to change in \( V_{th} \) computed at the nominal \( V_{th} \). Substituting \( \Delta V_{th} \) from (2), we obtain:

\[
D(t) = D(0) + K f(t)
\]

We refer to the constant, \( K = S c \ h(\xi) \), as the \( K \)-value of the device, \( D(t) \), which combines the effects of temperature, voltage, and the stressing SP. We observe that under a fixed temperature, supply voltage, and SP, the gate delay increases monotonically with time.

### B. Initial ROSC calibration and aging estimation

Fig. 4 depicts an example where a CUT has four near-critical paths, each of whose temporal delay shifts are of the form (3). The paths may have different \( K \)-values, and some may become critical during some time period. The delay of the circuit, \( D_{\text{CUT}}(t) \), is the envelope of these delay curves, and may have points of nondifferentiability where the critical path changes, as shown by the encircled points in the figure. Thus, \( D_{\text{CUT}}(t) \) is characterized by a set of \( K \)-values, one for each differentiable segment of the curve. In contrast, the ROSC has a single critical path with a single \( K \)-value, \( K_{\text{ROSC}} \), that is typically different from any of the values for the CUT.

At the presilicon stage, \( D_{\text{CUT}}(t) \) can be obtained by performing STA on the CUT at multiple points during its lifetime under the worst-case stressing probabilities, hence alternatively called \( D_{\text{run}}(t) \) in Fig. 4. A simpler and accurate representation for the CUT delay, which we use in this work, was introduced in [15] using a tight upper-bound on \( D_{\text{CUT}}(t) \), denoted by \( D_{\text{UofM}}(t) \) and shown in Fig. 4. For \( t \in [t_0, t_f] \), this bound is given by:

\[
D_{\text{UofM}}(t) = D_{\text{CUT}}(t_0) + K_{\text{CUT}}(t_f - t_0)
\]

where \( K_{\text{CUT}} = \frac{D_{\text{CUT}}(t_f) - D_{\text{CUT}}(t_0)}{t_f - t_0} \)

This bound is characterized by a single \( K \)-value, \( K_{\text{CUT}} \), throughout the lifetime of the CUT, and can be computed using just two STA runs. The values of \( D_{\text{CUT}}(t_0) \) and \( D_{\text{CUT}}(t_f) \) are obtained by performing STA on the CUT at \( t_0 \) and \( t_f \), respectively, assuming the worst-case workload on the CUT. Setting \( D_{\text{CUT}}(t) \) to this bound,

\[
D_{\text{UofM}}(t) = D_{\text{CUT}}(t_0) + K_{\text{CUT}}(t_f - t_0)
\]

the delay degradations of the CUT and the ROSC at \( t \in [t_0, t_f] \) are:

\[
\Delta D_{\text{CUT}}(t) = K_{\text{CUT}}(f(t) - f(t_0))
\]

\[
\Delta D_{\text{ROSC}}(t) = K_{\text{ROSC}}(f(t) - f(t_0))
\]

We define a calibration factor, \( D \), for the ROSC as

\[
D = \frac{\Delta D_{\text{CUT}}(t)}{\Delta D_{\text{ROSC}}(t)} = \frac{K_{\text{CUT}}}{K_{\text{ROSC}}}
\]

The ROSC delay (i.e., the inverse of its oscillating frequency) reflects the delay degradation in the inverters in the ROSC. It is relatively easy to periodically monitor frequency degradation of the ROSC using the concept of beat frequencies [2] during runtime. Based on this ROSC measurement, (7) is used to predict the CUT delay from the ROSC delay. By placing the ROSC close enough to the CUT, it may experience similar process variations, and undergo similar temperature and voltage stress, as the CUT. This is shown in [15] to keep the value of \( D \) constant over such variations.

### IV. Post-silicon Aging Estimation

In our scheme, \( D \) and CUT delays at the measurement instants are stored in an LUT, as shown in Fig. 3. The initial \( D \) is obtained from presilicon analysis under the worst-case stress assumption, and is used to translate the delay degradation in the ROSC to that in the CUT until the first measurement instant. At each measurement instant, the CUT delay is measured, providing an accurate view of the actual stressing conditions that it experiences, and \( D \) is recalibrated. The LUT is then updated with the measured delay and the new \( D \). We now explain the theory behind the recalibration procedure, developing a new result in Theorem 1 that generalizes the presilicon upper bound of [15] to our postsilicon approach based on measurement and recalibration.

The input to the procedure is a set, \( T_M \), of \( N + 1 \) time instants, \( \{t_0, t_m, \ldots, t_{mN}, t_N\} \), where \( t_m \) is the \( i^{th} \) measurement instant, \( i = 1 \cdots (N - 1) \), \( t_0 = 0 \), and \( t_N = t_f \), and is known before manufacturing. In addition, presilicon analysis provides the worst-case delay of the CUT, \( D_{\text{wc}}(t) \), at these measurement instants, i.e., \( D_{\text{wc}}(t_m) \), by performing \( (N + 1) \) STA runs. We can also use \( D_{\text{UofM}}(t_m) \) instead of \( D_{\text{wc}}(t_m) \), which would require only two STA runs as explained in Sec. III-B, and reduce presilicon computation at the cost of accuracy.3 The knowledge of the near-critical paths of the CUT under the worst-case workload is also available from the presilicon analysis.

#### A. CUT delay estimate post-measurement

We define a factor, \( K_{\text{m}} \), as the maximum among the \( K \)-values (Sec. III-B) of the near-critical paths under the worst-case aging, i.e.,

\[
K_{\text{m}} = \max_{i \in \mathcal{S}_{\text{NC}}} \left[ \frac{D_{\text{wc}}(t_i) - D_{\text{wc}}(t_0)}{f(t_f) - f(t_0)} \right]
\]

where, \( \mathcal{S}_{\text{NC}} \) is the set of near-critical paths, and \( D_{\text{wc}}(t_i) \) and \( D_{\text{wc}}(t_0) \) are the delays of the \( i^{th} \) such path at \( t_0 \) and \( t_f \), respectively. In addition to the runtime delay measurement, we use \( K_{\text{m}} \) to estimate delay of the CUT. The rationale behind using \( K_{\text{m}} \) is that it is the largest among \( K \)-values of the near-critical paths under both the worst-case and realistic workload. Therefore, if the post-measurement delay estimate upper-bounds any path with \( K \)-value less than or equal to \( K_{\text{m}} \), it is guaranteed to cater to the absolute worst-case future workload of the CUT. The estimated delay \( D_{\text{est}}(t) \), is obtained based on Theorem 1 (proof deferred to the Appendix).

2BTI exhibits partial recovery during which \( V_{th} \) is slightly reduced. Equation (2) represents the envelope of degradation, which increases monotonically.

3Loss of accuracy is minimal as \( D_{\text{UofM}}(t) \) is a tight upper-bound [15].
The above equations are executed recursively at every \( t \) as explained in Sec. I to obtain and recalibration were not performed.

\[ \Delta D(t) = D_{act}(T_M(c)) + K_{min}(f(t) - f(T_M(c))) \]  
\[ K_{min} = \min \left( K_{f}, \left( \frac{D_{act}(T_M(c+1)) - D_{act}(T_M(c))}{f(T_M(c+1)) - f(T_M(c))} \right) \right) \]

The above equations are executed recursively at every \( T_M(c) \), from \( c = 0 \) to \( N - 1 \). Then \( D_{act}(t) \) is an upper-bound on the actual delay of the CUT under every possible realistic workload.

Theorem 1 provides a single \( K \)-value, \( K_{min} \), of the aging curve of the CUT between each set of consecutive measurement instants. Referring to Fig. 2, \( K_{min} \) is the the \( K \)-value of the delay estimate, \( D_{act}(t) \), beyond \( t_{m1} \), based on the measured CUT delay, \( D_{act}(t_m) \).

The first argument of the min function in (11) recalibrates the \( K \)-value. In theory, it is possible for this recalibrated equation to exceed the \( D_{act}(t_m) \) bound at some time instants, although this does not happen in our experiments. The second argument of the min function ensures that \( K_{min} \) can never exceed the \( D_{act}(t_m) \) bound.

Fig. 5 shows an application of Theorem 1 with three random measurement instants for \( wb \_dma \). We simulate a realistic workload as explained in Sec. I to obtain \( D_{act}(t_m) \), \( i = 1, \ldots, 3 \). In this circuit, both the UofM bound and pessimistic trajectory, \( D_{act}(t) \) are the same due to a dominant critical path. Hence the estimated delay, \( D_{act}(t) \) follows \( D_{wc}(t) \), until \( t_{m5} \). Beyond \( t_{m5} \), \( D_{act}(t) \) is updated based on the true measured delays. The dotted lines show the trajectories \( D_{act}(t) \) would have followed if further measurement and recalibration were not performed.

**C. Effect of PVT variations**

For correct functioning of the proposed framework, the sensors should try to match the process parameter variations, \( V_{dd} \), temperature, and the signal stress probabilities in the CUT. Due to spatial proximity of the ROSCs and the CUT, they face similar temperature stress, and systematic variations within the CUT in any manufactured part are similar to those in the ROSCs close to it. This proximity also enables the ROSCs to connect to the supply lines of the CUT thus capturing the effects of \( V_{dd} \) variations under DVFS and power gating.

These factors are incorporated automatically during presilicon analysis (refer Sec. III-B and proof of robustness to PVT variations in [15]). However, the random variations and the effect of stressing SPs (shown to be significant in Fig. 1) in the CUT are difficult to match with those of the ROSC from the presilicon analysis alone. Our approach captures these variations through direct measurement on the CUT at the measurement instants and use of \( D \). In particular, since the calculation of \( K_{min} \) integrates the effects of all near-critical paths (which could be potential critical paths in different manufactured parts), our bound in Theorem 1 is robust to all variations.

**V. EXPERIMENTAL SETUP AND RESULTS**

**A. Experimental setup**

The ideas in this paper are exercised on a set of representative IWL5 2005 benchmarks. The circuits are synthesized in Synopsys Design Compiler using the NanGate 45nm Open Cell Library barring the XOR, XNOR and specialized gates (half and full-adders, full-cells, antennae, tristate gates, and multiplexers). The circuits are aggressively optimized for timing during synthesis. We present our analysis based on the RD model of BTI, and select the value of \( c \) in (2) such that there is 25% degradation in PMOS \( V_{th} \) (due to NBTI) in 10 years [11]. The degradation in NMOS \( V_{th} \) due to PBII is assumed to be one-third of that due to PMOS NBTI [11].

We perform the simulations at the typical process and the worst-case temperature and voltage corner as specified by the NanGate library, i.e., at \( T = 125^\circ C \) and \( V_{dd} = 0.9 \) V. This choice is not critical as our method is robust to variations as explained in Sec. IV-C. The beginning \( t_0 \) and end of lifetime \( t_f \) are zero and 10 years, respectively.

The presilicon analysis of each CUT requires two STA runs (at \( t_0 \) and \( t_f \), the runtime of which is less than a minute even for the largest benchmark circuit. To simulate the long-term realistic operating environment of the CUT, we choose a distribution of signal probabilities as in the description of Fig. 1.

**B. Choice of \( N \) and \( t_m \)**

Since BTI-induced aging is proportional to a sub-linear function of time, \( f(t) \), the maximum degradation occurs towards the beginning of lifetime. Hence the interval between the measurement instants should be chosen linearly in \( f(t) \) for the best post-silicon aging estimation.

Under the RD model assumed here, for \( N \) measurement instants, we chose the \( i^{th} \) measurement instant, \( t_{m_i} \), \( i = 1, \ldots, N \) as:

\[ t_{m_i} = \left( \frac{i}{M + 1} \right)^{1/n} t_f \]

where \( M \) is increased from one, until the first \( N \) non-negligible values of \( t_{m_i} \) are obtained. Each \( t_{m_i} \) is rounded off to the nearest half-year for convenience. We present the results for \( N = 1, 2, 3, 4 \), which correspond to the four sets of \( t_{m_i} \) values.

**C. Effects of recalibration**

We begin with a single circuit, \( wb \_dma \) for recalibration at a single measurement instant, \( t_m \). Using five sets of SPs from \( t \in [0, t_m] \), we obtain five realistic aging curves for the CUT until \( t_m \). For each such
curve, we apply ten different sets of SPs from \( t_m \) to \( t_f \), to obtain a total of 50 realistic workload scenarios. If \( i \) indexes the five SP sets from 0 to \( t_m \), and \( j \) indexes the ten SP sets from \( t_m \) to \( t_f \) for each \( i \)th set, we obtain five groups of ten actual delay curves.

We define the average speed wastage factor, \( \text{SWF}(k) \) for a particular type of workload (indexed by \( k \)), as the time-average of \( \text{SWF}(t_f,k) \) defined in (1). We plot \( \text{SWF}(k) \) for the above 50 realistic workload scenarios (i.e., \( k = 1, \ldots, 50 \)) vs. the location of \( t_m \) in Fig. 6. For example, the first set of ten bars for \( t_m = 0 \) represents the ten workloads corresponding to \( i = 1 \) and \( j = 1, \ldots, 10 \), the second set represents the next ten workloads for \( i = 2 \) and \( j = 1, \ldots, 10 \) and so on. Clearly, the goodness of estimation increases initially as \( t_m \) is increased as shown by the reduced \( \text{SWF}(k) \), but the advantage of the recalibration scheme slowly reduces as we keep shifting \( t_m \) towards \( t_f \). In fact, \( \text{SWF}(k) \) is reduced uniformly over all \( k \) irrespective of the actual workload of the CUT when \( t_m \sim 1 \) year (our choice of \( t_m \) for \( N = 1 \) in (V-B) is consistent with this observation), whereas for \( t_m = 0 \) or close to \( t_f \), reduction in \( \text{SWF}(k) \) depends on the workload. This is attributed to the fact that BTI is a front-loaded process, and if the aging trend in the CUT is captured early in time (see Sec. V-B), its future aging trend can also be optimally captured by our approach, irrespective of the true workload.

Fig. 6. Average speed wastage, \( \text{SWF}(k) \), vs. temporal shift in \( t_m \) in \text{wb_dma}.

Next, we use the MC simulations as described in Sec. I to obtain the statistics of the \( \text{SWF}(t_f) \) (defined after (1)) for multiple CUTs with \( N \) measurement instances whose locations are chosen as (15). Without any post-silicon measurement, each element of \( \text{SWF}(t_f) \) is large as shown by their mean and maximum for the \( N = 0 \) case\(^4\) in Fig. 7 as this is based on the worst-case aging scenarios. Both mean and range of \( \text{SWF}(t_f) \) decrease with increasing \( N \) as the real aging trends of the CUT are captured by the true delay measurements.

Let us now look at the data from Fig. 7 in more details and focus on the error in \( \Delta D_{act}(t) \) instead of \( D_{act}(t) \) (which was incorporated in \( \text{SWF}(k,t) \)). We quantify the error in \( \Delta D_{act}(t) \) by the vector, \( E \), whose \( i \)th element, corresponds to the \( i \)th MC run, and defined as:

\[
E(i) = \text{Mean}_t \left[ \frac{\Delta D_{act}(t) - \Delta D_{est}(t)}{\Delta D_{act}(t)} \right] \cdot t > 0
\]

where \( \Delta D_{act}(t) \) and \( \Delta D_{est}(t) \) are the estimated and actual delay degradation of the CUT from \( t = 0 \), respectively, at the \( t \)th run, and the parenthetical expression in (16) is averaged over \( t \) to obtain \( E(i) \). Table I reports the statistics of \( E \) in terms of its mean and range (minimum and maximum) with \( N \). The first column denotes the CUT. The second to fifth columns are each divided into three sub-columns representing the minimum, average, and maximum values of \( E \) for \( N = 0, 1, \ldots, 4 \), respectively. The columns also show the vector of measurement instants, \( T_M \) (defined in Sec. IV), in years, excluding \( t_0 \) and \( t_f \) in years, for each \( N \) based on (15).

As observed in Fig. 5, there is large difference between the worst-case delay curve and the actual delay. This difference varies from circuit to circuit, based on the topology, depth and structure of the near-critical paths. Hence the error in \( \Delta D_{act}(t) \) is very high without any post-silicon calibration (\( N = 0 \) case) as seen in Table I. This error is reduced as \( N \) is increased (as seen in both Fig. 5 and Table I), since the true delay measurement of the CUT incorporates information about its past aging scenario to bring down the pessimism. However, pessimism still exists due to the worst-case SP approximation to derive the \( K \)-values beyond each measurement instant, because of which the error is always non-zero even after increasing \( N \).

We have also observed that by selecting the measurement instants as (15), we achieve the best results. Any other schedule of selecting these potentially increases the error even if \( N \) is increased.

D. Contents of the LUT

The circuits fall in two categories: ones where the multiple critical paths “cross over” as shown in Fig. 4, and others in which the path critical at \( t_0 \) remains critical through \( t_f \), among a set of near-critical paths. For the CUTs in first category, the \( D \) values are updated by our proposed methodology, whereas for the second, they remain fairly constant after every measurement instant. However, according to [15], for the current library, the amount of crossover between paths as shown in Fig. 4 is minimal, and the upper-bounded curve is very close to the maximum delay curve. We observed a similar case and most of the CUTs in Table I belonged to the second category in spite of our aggressive timing optimization. Hence, after each measurement instant, barely any change was observed in \( D \) values of the CUTs, and the LUT was updated only with the measured CUT delays.

VI. CONCLUSION

We have proposed an algorithm to reduce pessimism in estimating BTI-induced aging in digital circuits in terms of their delay degradation. Our estimation scheme is facilitated by both post-silicon runtime measurements and updates to the sensor calibration factor.

APPENDIX I

In this section, we present a proof of Theorem 1. We begin by presenting a lemma [15] that provides a tight upper-bound on the maximum of a set of monotonically increasing curves:

**Lemma 1:** In the interval \([t_0, t_f]\), an upper-bound on the maximum of a set of monotonically increasing functions \( x_1(t), x_2(t), \ldots, x_{n+1}(t) \) such that \( x_i(t) = x_i(t_0) + \int_{t_0}^{t} (f(t) - f(t_0)) \) is given by:

\[
y_n(t) = x_M(t_0) + \int_{t_0}^{t_f} \left[ \frac{x_M(t_f) - x_M(t_0)}{f(t_f) - f(t_0)} \right] (f(t) - f(t_0))
\]
where the function \( x_M(t) = \max_{t \in [0, t]} x(t) \) represents the upper envelope of the functions \( x(t) \) for \( t \in [0, t_N] \).

Referring to Fig. 3 and (3), the path delays can be expressed as \( x(t) \). The maximum of the path delays is the delay of the CUT represented by the piece-wise smooth curve \( D_{ucf}(t) \) in Fig. 4. Lemma 1 provides the upper-bound, \( D_{ucf}(t) \) on \( D_{c UT}(t) \) derived in a similar fashion as the \( y_n(t) \) for the maximum of the \( x_i(t) \).

Proof of Theorem 1: Based on (3), delay of any near critical path, \( p_i \), under realistic workload, can be expressed as \( D_{p_i}^D(t) = D_{pc}^D(t) + K_{p_i}^M(t) \), such that the actual delay of the CUT is the maximum over such path delays, i.e., \( D_{ucf}(t) = \max_{p_i} D_{p_i}^D(t) \). Similarly, delay of the same path under the worst-case workload can be expressed as \( D_{ucf}^M(t) \), such that \( D_{ucf}(t) = \max_{p_i} D_{p_i}^M(t) \). This is represented by \( D_{ucf}^M(t) \) in Fig. 4.

From Lemma 1, we only need \( D_{ucf}(t) \) and \( D_{ucf}^M(t) \) to obtain the upper-bound delay curve with a single K-value. Since there is no aging at \( t = 0 \), \( D_{ucf}(0) = D_{ucf}^M(0) \), which also implies that \( D_{ucf}(t) = \max_{p_i} D_{p_i}^M(0) \). Similarly, the worst-case delay is always more than the actual delay because of which \( D_{ucf}^M(t) \) is always a lower bound on \( D_{ucf}(t) \).

Keeping the above conclusions in mind, we now begin the proof formally. We use mathematical induction on the number of measurement instants, \( N \), to show that \( D_{ucf}(t) \geq D_{p_i}^D(t) \), \( \forall p_i \), and \( t \in [0, t_f] \).

**Basis case:** For \( N = 1 \), \( t_M = [0, t_f] \) and \( D_{ucf}^M(0) = D_{ucf}(0) \). Since by definition of \( K_{p_i}^M(t) \), \( D_{ucf}(0) + K_{p_i}^M(t_f) \geq D_{ucf}^M(t_f), \) for \( t \in [0, t](11) \)

\[ D_{ucf}(t_f) = D_{ucf}(0) + \sum_{t \in [0, t_f]} K_{p_i}^M(t_f) \]

The \( D_{ucf}(t_f) \) is obtained using (10): as \( t \in [0, t_f] \).

\[ D_{ucf}(t_f) = D_{ucf}(0) + \sum_{t \in [0, t_f]} K_{p_i}^M(t_f) \]

From Lemma 1, \( D_{ucf}(t_f) \) forms an upper-bound on the maximum of path delays under worst-case workload. Since the realistic workload is always more relaxed than the worst-case one, \( D_{ucf}(t_f) \leq D_{p_i}^D(t_f) \), \( \forall p_i \) and \( t \in [0, t_f] \).

**Inductive hypothesis:** For \( N = r \), \( t_M = [0, t_m \ldots, t_n] \), and \( D_{ucf}^M(t) = D_{ucf}(t) \). Since by definition of \( K_{p_i}^M(t) \), \( D_{ucf}(t) + K_{p_i}^M(t_f) \geq D_{ucf}^M(t_f) \), for \( t \in [0, t](11) \)

\[ D_{ucf}(t_f) = D_{ucf}(0) + \sum_{t \in [0, t_f]} K_{p_i}^M(t_f) \]

The \( D_{ucf}(t_f) \) is obtained using (10): as \( t \in [0, t_f] \).

\[ D_{ucf}(t_f) = D_{ucf}(0) + \sum_{t \in [0, t_f]} K_{p_i}^M(t_f) \]

From Lemma 1, \( D_{ucf}(t_f) \) forms an upper-bound on the maximum of path delays under worst-case workload. Since the realistic workload is always more relaxed than the worst-case one, \( D_{ucf}(t_f) \leq D_{p_i}^D(t_f) \), \( \forall p_i \) and \( t \in [0, t_f] \).

**Inductive step:** For \( r = r + 1 \), \( t_M = [0, t_m \ldots, t_n, t_f] \), and \( t \in [0, t] \), we can ascertain that \( D_{ucf}(t) \) as defined in (10) forms an upper-bound on \( D_{p_i}^D(t) \) for \( t \in [0, t] \), and \( \forall p_i \) from the inductive hypothesis.

Now for \( t \in [t_m, t_f] \), depending on \( D_{ucf}(t_m) \) and \( K_{p_i}^M(t_f) \), \( K_{p_i}^M(t_f) \) is either \( K_{p_i}^M(t_m) \) or \( (D_{ucf}(t_f)-D_{ucf}(t_m))/(t_f-t_m) \) (Case-2).

**Case 1:** \( e_{11}(t) = D_{ucf}(t_f) - D_{p_i}(t_f) \). Then, \( e_{11}(t) = (D_{ucf}(t_m) - D_{p_i}(t_m)) + (K_{p_i}^M(t_f) - K_{p_i}^M(t_m))(f_f(t) - f(t_m)) \).

Each of the parenthetical expressions in \( e_{11}(t) \) is positive due to the following: 1. Due to monotonic property of \( f(t) \). 2. By definition, \( K_{p_i}^M(t_m) \geq K_{p_i}^M(t_f) \).

3. The actual delay \( D_{ucf}(t_m) \geq D_{p_i}(t_m) \), since \( D_{ucf}(t_m) \) was obtained on account of having maximum delay among all the near-critical paths.

**Case 2:** \( e_{21}(t) = D_{ucf}(t_f) - D_{p_i}(t_f) \). Then, \( e_{21}(t) = (D_{ucf}(t_m) - D_{p_i}(t_m)) + \frac{(D_{ucf}(t_f) - D_{ucf}(t_m)) - K_{p_i}^M(t_f) - K_{p_i}^M(t_m)}{t_f-t_m} \).

Since \( D_{ucf}(t) \) and \( D_{p_i}(t) \) increase monotonically, \( e_{21}(t) \) either monotonically increases or decreases in \( t \in [t_m, t_f] \). By algebraic manipulation,

\[ e_{21}(t) = D_{ucf}(t_m) - D_{p_i}(t_m) + \frac{(D_{ucf}(t_f) - D_{ucf}(t_m))}{t_f-t_m} - K_{p_i}^M(t_f) - K_{p_i}^M(t_m) \]